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ANALYTICAL APPROXIMATIONS

Volume 3

Cecil Hastings, Jr.

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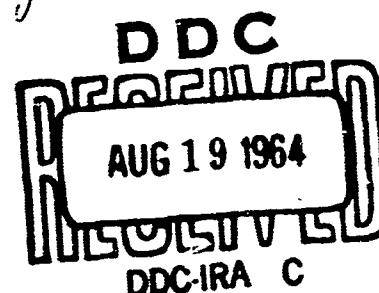
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Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R, x) = \int_R^{\infty} e^{-\frac{1}{2}(\rho^2+x^2)} I_0(\rho x) \rho d\rho$$

in which $I_0(z)$ is the usual Bessel function. To better than .00014 over $(-\infty, \infty)$,

$$q(1, x) \doteq 1 - \frac{.3935}{[1 + .3968x^2 + .0047x^4 + .00028x^6]^4}$$

The parametric form used is convenient for approximating fixed-R cross-sections of the $q(R, x)$ surface for small values of R.

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Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R, x) = \int_R^\infty e^{-\frac{1}{2}(\rho^2+x^2)} I_0(\rho x) / \rho d\rho$$

in which $I_0(z)$ is the usual Bessel function. To better than .0035 over $(-\infty, \infty)$,

$$q(2, x) \doteq 1 - \frac{.865}{[1 + 0.38x^2 + 0.001x^4]^L}$$

The parametric form used is convenient for approximating fixed-R cross-sections of the $q(R, x)$ surface for small values of R :

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Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R, x) = \int_R^{\infty} e^{-\frac{1}{2}(\rho^2 + x^2)} I_0(\rho x)^2 d\rho$$

in which $I_0(z)$ is the usual Bessel function. To better than .001 over $(-\infty, \infty)$,

$$q(2, x) \doteq 1 - \frac{.865}{[1 + 0.0401x^2 + 0.00309x^4 + 0.000075x^6]^4}$$

The parametric form used is convenient for approximating fixed-R cross-sections of the $q(R, x)$ surface for small values of R.

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Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R, x) = \int_R^{\infty} e^{-\frac{1}{2}(\rho^2 + x^2)} I_0(\rho x) / \rho d\rho$$

in which $I_0(z)$ is the usual Bessel function. To better than .0014 over $(-\infty, \infty)$,

$$q(1, x) \doteq 1 - \frac{.393}{[1 + .093x^2 + .007x^4]^4}$$

The parametric form used is convenient for approximating fixed-R cross-sections of the $q(R, x)$ surface for small values of R.

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Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R, x) = \int_R^\infty e^{-\frac{1}{2}(\rho^2+x^2)} I_0(\rho x) d\rho$$

in which $I_0(z)$ is the usual Bessel function.

To better than .0009 over $(0, \infty)$,

$$q(2, 2+y) \approx 1 - \frac{.397}{[1 + .236y + .066y^2 + .056y^3]^4}$$

The parametric form used is convenient for approximating fixed-R semi-cross-sections of the $q(R, R+ty)$ surface for any $R > 0$ and for y ranging over $(0, \infty)$.

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